Locally Weighted Scatterplot Smoothing (LOWESS)
The classical procedure to smooth a scatterplot is to fit a polynomial of suitable degree. The problem with polynomial smoothing is that it is not resistant. A few data points at the extreme right of the scatterplot can very much affect the fitted values at the left of the scatterplot. In fact, this is a general problem of fitting curves with the least squares method. Lowess regression, introduced by Cleveland (1979), is based on a smoothing procedure which pays greater attention to the local points. The smoothed value of y corresponding to a data point \( x_i \), say, is obtained on the basis of the data points around it within a band of certain width. The point \( x_i \) is the midpoint of the band. The data points within the band are assigned weights in a way so that \( x_i \) has the highest weight. The weights for the other data points decline with their distance from \( x_i \) according to a weight function. The weighted least squares method is used to find the fitted value corresponding to \( x_i \), which is taken as the smoothed value. The procedure is repeated for all the data points.

**Box 4.1: Computation of lowess**

1. Choose a fraction \( f \) of the data points which is to be used for computation of each fitted value. Let \( b \) be the nearest integer to \( f.n/2 \) where \( n \) is the number of all data points i.e. size of the data. In other words, \( 2b \) is the number of points around each \( x \) (including itself) i.e. the bandwidth, to be used for fitting. In practice, you often have to try out a few choices of \( f \), say between 0.4 to 0.8. Greater the value of \( f \), greater the smoothing effect but less may be details of the curve.

2. Let \( d_i \) be the distance from \( x_i \) to its \( b \)th nearest neighbour along the x axis and \( T \) be the weight function. Then the weight \( w_k \) given to the point \( (x_k, y_k) \) when computing a smoothed value at \( x_i \), is as follows:

\[
w_k = T\left( \frac{x_i - x_k}{d_i} \right) \quad \text{where} \quad T(u) = \begin{cases} (1-|u|)^3 & \text{for } |u| < 1 \\ 0 & \text{otherwise} \end{cases}
\]

If \( d_i \) is 0, meaning that the \( b \) nearest neighbours of \( x_i \) all have abscissas equal to \( x_i \), then points whose abscissas are equal to \( x_i \) are given weight 1 and all other points are given weight 0. In this a constant is fit instead of a line.

3. To compute the fitted value at \( x_i \) (or constant when \( d_i \) is 0) a weighted least squares fit is obtained. Thus, we have –

\[
b_{\text{estimate}} = \frac{\sum w_i^2 (x_k - \bar{x})(y_k - \bar{y})}{\sum w_i^2 (x_k - x)^2} \quad \text{and} \quad a_{\text{estimate}} = \bar{y} - b_{\text{estimate}} \bar{x}
\]

where \( \bar{y} \) and \( \bar{x} \) are weighted means

fitted \( y_i = a_{\text{estimate}} + b_{\text{estimate}} \cdot x_i \)